Summer 2016
Lesson Plan Project for Number Theory

Geometry lessons, 7th and 8th grade lessons and Algebra lesson

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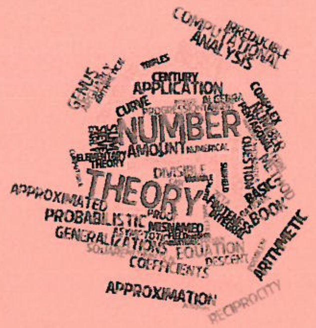


What is Number Theory?

Number theory, also known as higher arithmetic, is a branch of mathematics concerned with the properties of integers , rational numbers, irrational numbers, and real numbers. Sometimes the discipline is considered to include the imaginary and complex numbers as well.

Formally, numbers are represented in terms of set s; there are various schemes for doing this. However, there are other ways to represent numbers -- for example, as angles, as points on a line, as points on a plane, or as points in space.

Our project covers some number theory topics form the junior high through the senior high level. We will look at what "perfect" numbers are as well as "abundant" and "deficient" numbers which are all defined by the sum of their factors. **Hot and Cold Cubes** will be used as a framework for operations with integers, and the **20 Question Game** will take us through other ways numbers are identified. We will then look at patterns in triangular and square numbers before getting into the discovery of trigonometric ratios. The last topic will focus on the irrational number known as the "golden ratio" and look at how this ratio is found in art architecture and nature, including the human body.

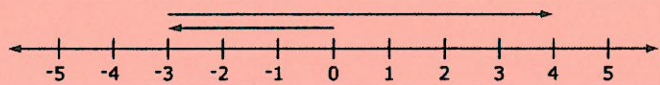
Sample MCA Test Questions (Grade 7)

Which numbers are rational? Select the numbers you want.

a. 2   b. $1\frac{7}{8}$ c. π d. $\sqrt{5}$ e. $6.3\overline{9}$

Which shows a model of $-3 + 4$?

A.



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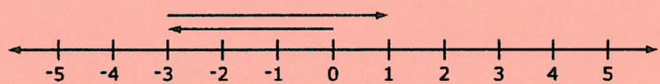
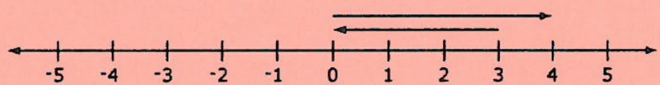
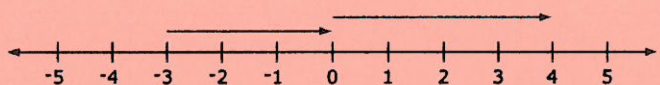


C.



D.



An equation is shown.

$$n = 1 \div 17$$

Which describes *n*?

A. Integer

B. Irrational

C. Rational

D. Whole

Name____________________

Number Theory Pretest

1. List the factors of 42.

   Is 42 a perfect, deficient or abundant number?

2. Give an example of 3 perfect numbers.

3. What is the property that all perfect numbers possess?

4. Give one example of the "golden ratio" existing in:
   a. Art
   b. Architecture
   c. Nature

5. Given the categories a. Prime, b. Composite, c. Natural, d. Whole, e. Integer, f. Rational, g. Irrational. For each number list all the categories that each number belongs to.
   1. -7
   2. 0
   3. .42
   4. 11
   5. $\sqrt{36}$
   6. $\sqrt{2}$
   7. $\frac{8}{9}$

6. Describe 3 ratios from the human body that may possess the golden ratio.
   1.

   2.

   3.

7. Find the sums

-7+-8 $\quad\quad$ $\frac{2}{3} + -\frac{1}{6}$ $\quad\quad$ -3.8+2.7

8. Complete each number sentence

-532 ÷ (-7)= -12*-23=

9. In a football game, one team make seven plays in the first quarter. The results of those plays in order: gain of 7 yards, gain of 2 yards, loss of 5 yards, loss of 5 yards, loss of 12 yards, gain of 15 yards, gain of 8 yards, lose of 8 yards.

a.) what is the overall gain (or lose) from all seven plays?

b.) What is the average gain(or lose) per play?

Minnesota standards:

____ 9.3.4.1 <u>Understand how the properties</u> of similar right triangles allow the trigonometrios ratios to be defined, and determine the sine, cosine and tangent of an acute angle in a right triangle.

____ 9.3.4.2 <u>Apply the trigonometric ratios</u> sine, cosine and tangent to solve problems, such as determining <u>lengths and areas in right triangles and in figures that can be decomposed into right triangles</u>. Know how to use calculators, tables or other technology to evaluate trigonometric ratios.*For example*: Find the area of a triangle, given the measure of one of its acute angles and the lengths of the two sides that form that angle.

Building bridges Day 1 trig

This activity encourages students to discover the importance of triangles in real-life constructions. Students build bridges, using a limited supply of resources, and test the bridge's ability to hold weight.

Goals:
Classifies, constructs and determines the properties of triangles and quadrilaterals. Analyses a mathematical or real-life situation, systematically applying problem-solving strategies

Materials:
• 6 drinking straws (preferably the straight plastic variety, not the bendy type)
• 2 sticky labels • 2 maths textbooks • a ruler per group • a pair of scissors per group • one set of small graduated masses (1g – 200g) (I use science teacher)

<u>Launch:</u>
Using photographs of appropriate bridges in the local area
• Ask students to view the bridge photographs then visualise the bridges they know.
• In groups of 3 or 4, students have 5 minutes to discuss the various features of these bridges.
Suggested Questions
• What do the bridges look like? • What features do the bridges have in common?
• What two dimensional shapes have occurred repeatedly in these bridge constructions?

<u>Explore/Share:</u>
• In groups, students have 30 minutes to complete the bridge building task (Tasksheet building bridges)
• Bridges are load tested and results recorded
Check that students: • understand the task • are only using the given resources • understand how the bridge will be tested with the masses • As you have limited resources to build the bridge, what strategies might you use to ensure you do not waste materials?

<u>Summarize:</u>
• Whole class discussion to close lesson • Outline the purpose of the activity: to

look at why triangles are often used in constructions. Triangles are at the heart of our trig. Unit. What have you learned while building this bridge? Where else do you see triangles?
Home work:

What is the problem?

You are in a group which is to abseil down a rock face tomorrow. Your task is to estimate the height of the face. You have no measuring instruments. You need to determine the height to know how much rope to take. You cannot take excess rope as you are at the start of a four day exercise and you must not have extra weight with you. Tomorrow morning you will walk the trail which will take you to the top of the rock face.

Brainstorm as many ways as possible to estimate the height of the rock face. Record all ideas, even if they appear absurd. You will share with group then, Each group will share their ideas with the class.

Trigonometry lesson day 2

This activity assesses a starting point for learning trigonometry. It encourages divergent thinking and attempts to address the reasons why trigonometry was developed. Students contribute their own solutions to problems before the introduction of scale drawings or trigonometric ratios.

Talk over Homework and put ideas on board.
Questioning
What information is available?
How could you measure the height?
What units of measure could you use?
How accurate would this method be?
What problems may arise using the method you have described?
What is another way of measuring the height?
Summerize:

• Whole class discussion on the accuracy and limitations of the different ways ofcalculating the height. How do we know?

Same shape triangles

In this activity students use practical measurement skills and ratio calculations to find a pattern linking the ratio of sides of a triangle with the angles. This lesson is designed to develop the concepts of sine, cosine and tangent ratios of angles.

Students should: I take the rest of day 2 to review this with my 10th grade geometry students
1. have a knowledge of the concept of ratio
2. have the ability to convert fractions to
decimals using a calculator to 3 decimal places
3. be able to measure the length of sides of triangles to the nearest millimetre

Day 3

Goals:

Applies trigonometry to solve problems (diagrams given) including those involving angles of elevation and depression. Explains and verifies mathematical relationships.

materials

• pencil • ruler • calculator • scrap paper • one set of triangles per group. Triangle sheets A-G should be photocopied onto coloured cardboard. Carefully cut out the triangles and place each set (8 triangles in a set) into plastic bags.• One Calculating ratios for similar triangles worksheet per group • Three large charts (enlarge to A3) – one for each ratio (opp/hyp, adj/hyp and opp/adj) - for a class graph • At this point only label the graphs with the ratios – introduce the trig names sin,cos and tan at the end of this lesson

Launch:

Show the students a large right-angled triangle with one angle marked. Remind the students about the hypotenuse (from Pythagoras' theorem) and show them the opposite and adjacent sides in relation to the marked angles • Discuss the meaning of the words opposite and adjacent in this context

• Practice labelling right-angled triangles on the board. • Explain that the lesson involves investigations of the ratios of pairs of sides of right-angled triangles with angles of different sizes • Discuss the word ratio and what it means in this context. • Compare opp/hyp for a very large and a very small angle, as shown in diagram, and have students estimate which one will have the larger ratio

Questioning

• What is the hypotenuse of a right-angled triangle? Where do you find it? • What is ratio?

• What happens to the opp/hyp ratio when the angle is large?

• What happens to the opp/hyp ratio when the angle is small?

Explore/Share:

• Hand out one Calculating ratios for similar triangles worksheet and a set of triangles to each group

• Each student takes two triangles. They measure each side to the nearest millimetre and complete the worksheet for their triangles writing the ratios as a fraction and using a calculator to estimate them to 3 decimal places • Each group completes the worksheet including the mean values for each ratio to 2 decimal places • Members of the group stack their triangles as neatly as possible on top of each other and discuss their findings

Questioning

• What is the hypotenuse of a right-angled triangle? Where do you find it?• What is ratio? • What happens to the opp/hyp ratio when the angle is large? • What happens to the opp/hyp ratio when the angle is small?

Summarize:

• When every group worksheet is completed, one member of each group brings it forward with their group's stack of triangles and briefly reports their findings • Each group now plots its mean valueson the three class graphs. At this stage,• Class discusses graphs. Note the fact that triangles which have the same ratios also have the same angles. This is the basis for scale drawings where although the triangles are different sizes, the angles are in the same proportion or ratio. Explain to students that these ratios have special names:

opp/hyp is sine of the angle (sin)

adj/hyp is cosine of the angle (cos)

opp/adj is tangent of the angle (tan)

these ratios are used in a branch of mathematics called trigonometry or trig for short

Where have you heard the word trig before?

What information can you observe from each graph? They should be able to see that the ratio increases as the angle increases for the opp/hyp graph; the ratio decreases as the angle increases for adj/hyp and the ratio increases as the angle increases for opp/adj. What occupations use trigonometry in their jobs? All kinds of engineers, navigator, surveyor, architect, air traffic controller, cartographer, landscape architect, meteorologist, electronics designer, oceanographer, roofing contractor, marine engineer, geologist and sheet metal, heating and air-conditioning engineers.

Where is the word trigonometry derived? The word trigonometry is derived from two Greek words meaning 'triangle' and 'measurement'.

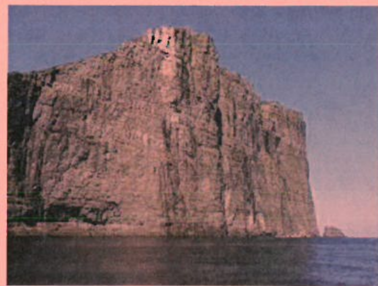
Building Bridge task
Day 1 of trig unit

Materials:
6 drinking straws (preferably the straight plastic variety, not the bendy type), 2 sticky address labels, 2 math textbooks, a ruler and a pair of scissors

Place the two textbooks (supports for the bridge) so that the distance between them is further than the length of one straw. These books represent the banks of a fast-flowing river, infested with leeches and people-eating crocodiles.
Your group must build a bridge to carry people from one side to the other.
You have 30 minutes to construct this bridge, using any of the given materials except the scissors and the ruler which may not be part of the bridge.
At the end of this time your bridge will be tested for strength and the results recorded.

The winning group will receive PBIS golden ticket certificates and bragging right with photos of their bridge.

Task 2 Your home work after day 1



What is the problem?

You are in a group which is to abseil down a rock face tomorrow. Your task is to estimate the height of the face. You have no measuring instruments. You need to determine the height to know how much rope to take. You cannot take excess rope as you are at the start of a four day exercise and you must not have extra weight with you. Tomorrow morning you will walk the trail which will take you to the top of the rock face.

Brainstorm as many ways as possible to estimate the height of the rock face. Record all ideas, even if they appear absurd. You will share with group then, Each group will share their ideas with the class.

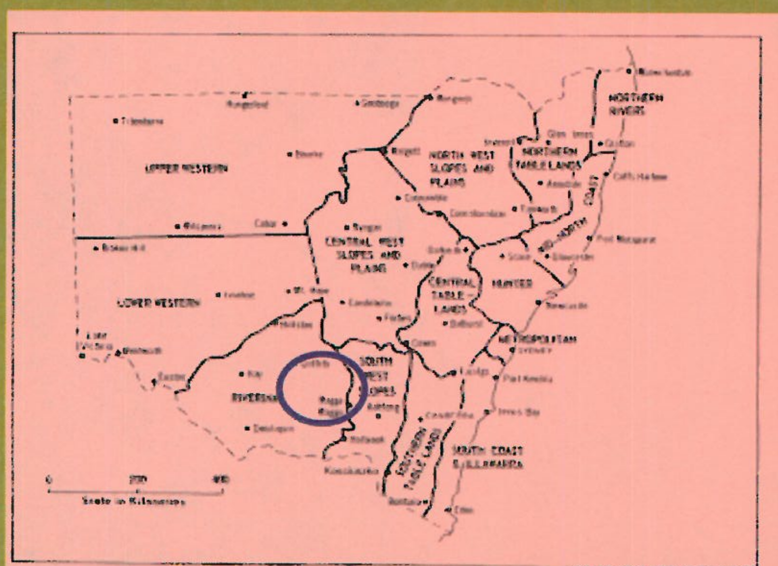
Building Bridges

Stage 5 Trigonometry

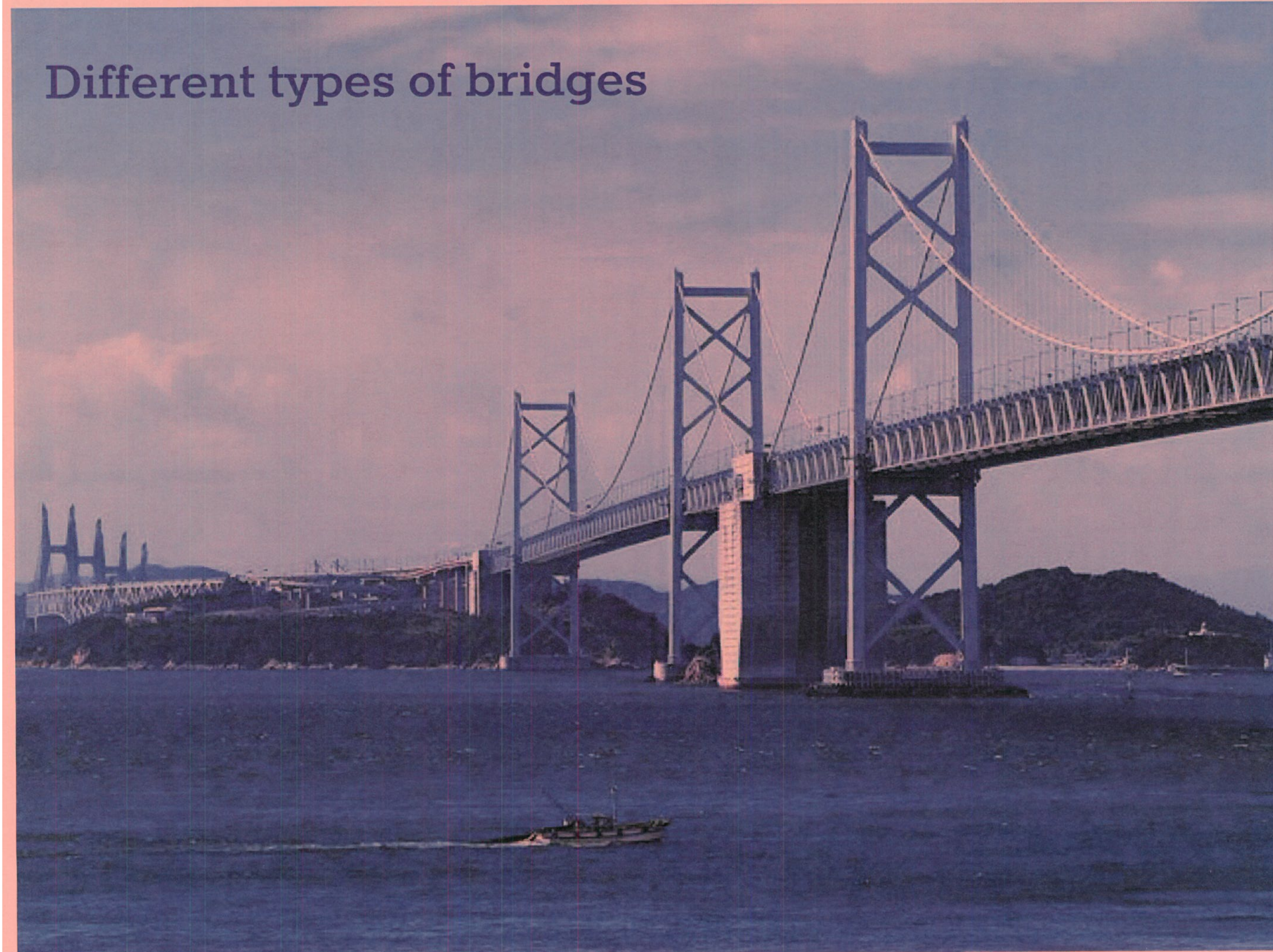
Mathematics, Curriculum K-12 Directorate

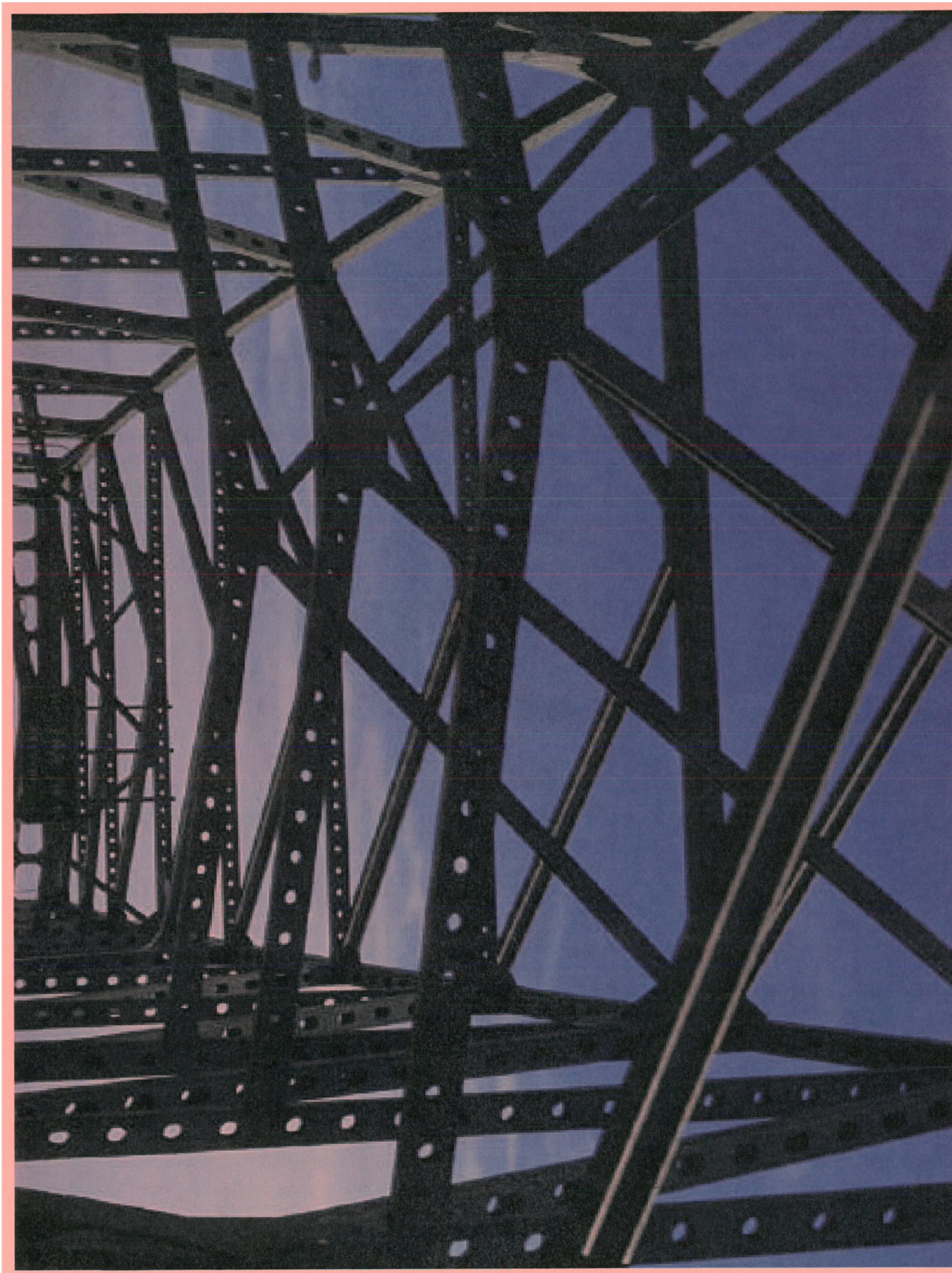
NSW Department of Education and Training

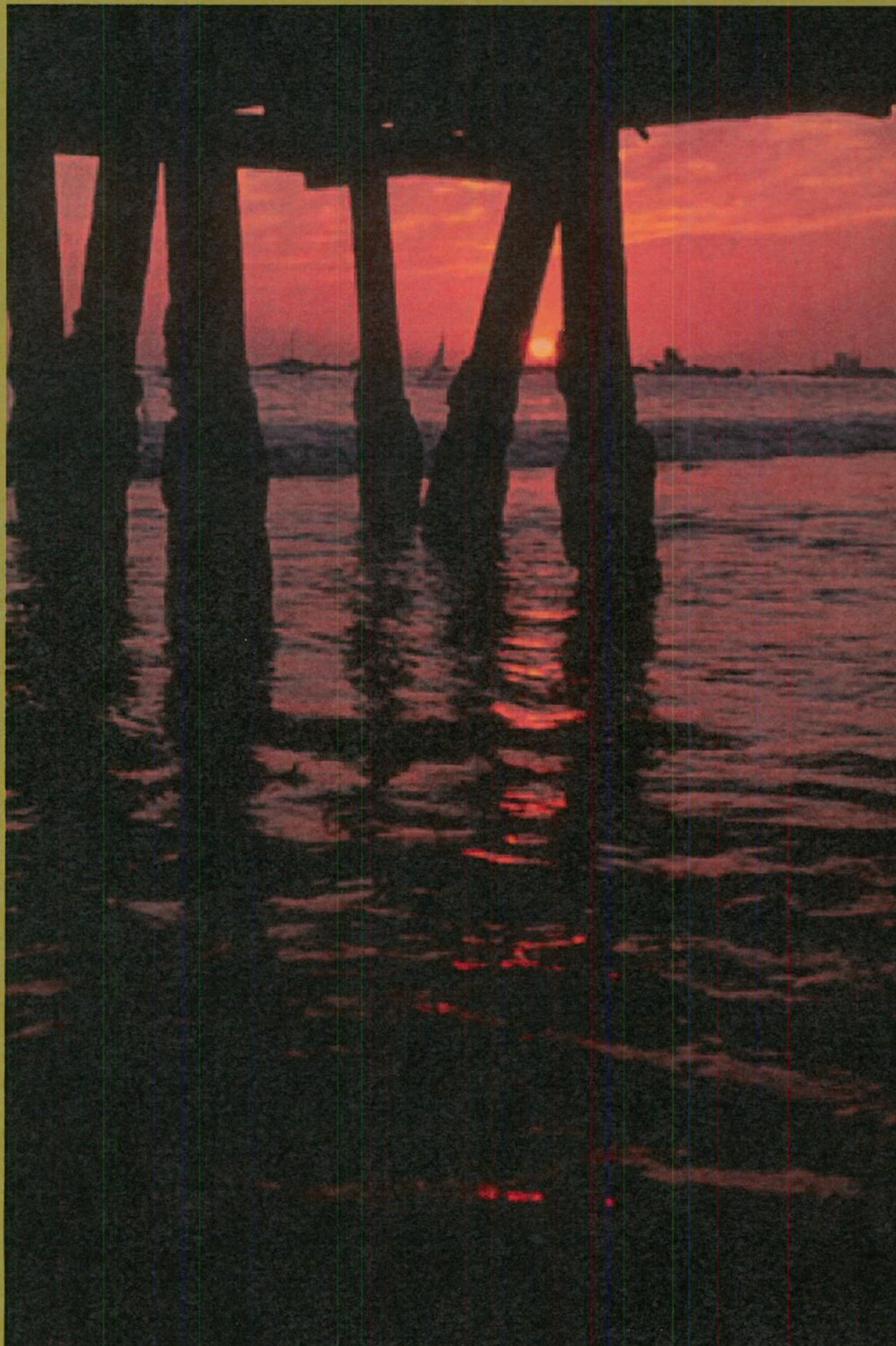
The Hampden Bridge



- Located over the Murrumbidgee River, Wagga Wagga, NSW
- Built in 1895
- 33.5 metre span timber truss bridge


Different types of bridges





What two dimensional shapes have occurred repeatedly in these bridge constructions?

Lesson: The Golden Ratio

Objective: Students will investigate and gather information about the "golden ratio" to develop an understanding of the topic. Students will be able to give examples from art, architecture and nature where the golden ratio exists.

Standard: Recognize proportional relationships in real-world and mathematical situations; represent these and other relationships with tables, verbal descriptions, symbols and graphs; solve problems involving proportional relationships and explain results in the original.

Launch: We will watch a short portion of the video "Donald Duck in Mathemagicland" that contains an introduction to the golden ratio. The golden ratio has also been a theme in some popular television shows and movies (CSI, Criminal Minds, Davinci Code) and we will use those examples as well to generate some interest in the topic. Students will be challenged to find something they think is "really cool" discovery about a golden ratio topic.

Explore: Students will work in pairs to investigate and develop an understanding of the golden ratio. They will be expected to write a brief report that includes their definition of the golden ratio along with examples of where the golden ratio exists in art, architecture, and nature

Share: Students will share their results with the teacher and the rest class and in particular their "really cool" discovery.

Summarize: The instructor will facilitate a discussion about the different areas where the golden ratio exists, in particular art, architecture, and nature.

Golden Ratio Report

Task Description:

1. Investigate and gather information for the terms Golden Ratio. Other terms closely related to the Golden Ratio include the Golden Rectangle, Golden Proportion, and Divine Proportion.

2. Write a report that includes information from your investigation. Your report should include, but is not limited to, the following:

   A. A personal understanding of the Golden Ratio should be apparent in the report (what is it?). **Make sure this is done in your own thoughts and words.**

   B. The Golden Rectangle or Golden Ratio should be correctly identified in several examples from:

      1. Art

      2. Architecture

      3. Nature

   C. Other topics to consider in your report might include how the Golden Ratio is related to work atributed to Pythagoras or Fibonacci.

Grading (25 points)

-Apparent understanding of topic (5 points)

-Examples identified in: Art (5 points)
Architecture (5 points)
Nature (5 points)

-Timeliness, overall writing of the paper (5 points)

****Bonus points may be earned for exceptional quality and detail.

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Lesson: "Are You Golden?" - The Golden Ratio

Objective: Students will investigate ratios within their own bodies to see if they possess any "golden" characteristics.

Standards: Recognize proportional relationships in real-world and mathematical situations; represent these and other relationships with tables, verbal descriptions, symbols and graphs; solve problems involving proportional relationships and explain results in the original.

Launch: I will ask the students to name, in their opinion, some of the most beautiful celebrities of this era. I will then introduce the concept of the golden ratio along with the theory that supposedly the most beautiful people and perfectly proportioned people possess the golden ratio throughout their facial and body features, which make them more appealing.

Explore: The students will work in pairs and make the following measurements on each other using a tape measure and record their data.

1. Belly button to floor and top of head to belly button.
2. Top of head to nose and nose to bottom of chin.
3. Belly button to bottom of knee and from bottom of knee to floor.
4. From top of head to bottom of neck and from bottom of neck to the belly button.
5. from top of head to bottom of chin and the width of the head (excluding ears).
6. 4-6 other ratios within the body or face that students think might be close to the golden ratio (some hints from the instructor may be necessary at this point).

Share: Students will share their results with the teacher and the rest class. discussion will come around to who thinks they have the most perfectly proportioned body.

Summarize: Students will write a brief summary of their findings in relation to their body ratios and will recognize that the golden ratio may or may not be a consistent ratio that exists in their body or any body. We don't want to make anyone feel inferior so we need to point out that the golden ratio being the ratio that makes a person's body perfectly proportioned it just a theory.

Get it? A **number** theory.

Measurement #1 (nearest 10th of cm)	Measurement #2 (nearest 10th of cm)	Ratio (large/small) -nearest 100th

The Chef's Hot and Cold Cubes

(Interactive Mathematics Program)

from: learn.shorelineschools.org/.../the_chefs_hot_and_cold_cubes11.doc

Numbers and Operations

7.1.2.1 Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive

OBJECTIVE:
Students will be able to add, subtract, multiply and divide positive and negative numbers.

LAUNCH
You may have learned some rules for doing arithmetic with positive and negative numbers. Many people find these rules hard to remember and don't understand where the rules come from. The following mythical story provides a context for understanding how positive and negative numbers work. Many people find it easy to remember the story many years after they first heard it, and the memory of the story enables them to reconstruct the rules. The story also helps some people make sense of the rules.

Read The Story and talk about each problem, we could use round robin reading like was model for us.

The Story

In a far-off place, there was once a team of amazing chefs who cooked up the most marvelous food ever imagined.

They prepared their meals over a huge cauldron, and their work was very delicate and complex. During the cooking process, they frequently had to change the temperature of the cauldron in order to bring out the flavors and cook the food to perfection.

They adjusted the temperature of the cooking either by adding special hot cubes or cold cubes to the cauldron or by removing some of the hot or cold cubes that were already in the cauldron.

The cold cubes were similar to ice cubes except that they didn't melt, and the hot cubes were similar to charcoal briquettes, except they didn't lose their heat.

If the number of cold cubes in the cauldron was the same as the number of hot cubes, the temperature of the cauldron was 0 degrees on their temperature scale.

For each hot cube that was put in the cauldron, the temperature went up one degree; for each hot cube removed, the temperature went down one degree. Similarly, each cold cube put in lowered the temperature one degree and each cold cube removed raised it one degree.

The chefs used positive and negative numbers to keep track of their changes they were making to the temperature.

For example, suppose 4 hot cubes and 10 cold cubes were dumped into the cauldron. Then the temperature would be lowered by 6 degrees altogether, since 4 of the 10 cold cubes would balance out the 4 hot cubes, leaving 6 cold cubes to lower the temperature 6 degrees. They would write

$$^{+}4 + {}^{-}10 = {}^{-}6$$

to represent these actions and their overall result.

Similarly, if they added 3 hot cubes and then removed 2 cold cubes, the combined result would be to raise the temperature 5 degrees. In that case, they would write

$$^{+}3 - {}^{-}2 = {}^{+}5$$

And if they wrote $^{-}5 - {}^{+}6 = {}^{-}11$, it would mean that first 5 cold cubes were added and then 6 hot cubes were removed, and that the combined result was to lower the temperature 11 degrees.

Sometimes they wanted to raise or lower the temperature by a large amount, but did not want to put the cubes into the cauldron one at a time. So for large jumps in temperature, they would put in or take out bunches of cubes.

For instance, if the chefs wanted to raise the temperature 100 degrees, then they might toss five bunches of 20 hot cubes each into the cauldron instead of 100 cubes one at a time. This saved a lot of time because they could have assistant chefs do the bunching.

When the chefs used bunches of cubes to change the temperature, they used a multiplication sign to record their activity. For example, to describe tossing five bunches of 20 hot cubes each into the cauldron, they would write

$$^{+}5 \cdot {}^{+}20 = {}^{+}100$$

where the $^+5$ meant that the five bunches were being added, and the $^+20$ showed that there were twenty hot cubes in each bunch.

The chefs could also change the temperature by removing bunches. For example, if they removed three bunches of 5 hot cubes each, the result was to lower the temperature 15 degrees, because each time a bunch of 5 hot cubes was removed, the temperature went down 5 degrees. To record this change, they would write

$$^-3 \cdot {}^+5 = {}^-15$$

where the $^-3$ meant that three bunches were being removed, and the $^+5$ showed that there were five hot cubes in each bunch.

EXPLORE

Have students work in groups. Each group should have a set of hot and cold cubes. One person in the group should be the recorder to write down the arrangements of the cubes (H for hot) (C for Cold) and the other person should be manipulating the cubes. Get the students started with the question...The chefs are starting with a cauldron that is at a temperature of 0. How can the chef make the temperature 2 degrees warmer? How many ways can you find? Let students see that you could use 2 hot cubes (H H) or could have many variations of H's and C's...HHHHCC. Then have the students make multiple ways to have the cauldron be a temperature of -2.

IMPORTANT:

Continue this until ALL of the students understand that there is more than one way to make a number.

1) Each of the problems below describes an action by the chefs. Figure out how the temperature would change overall in each of these situations and write an equation to describe the action and the overall result.

a) Three cold cubes were added and 5 hot cubes were added.
b) Five hot cubes were added and 4 cold cubes were removed.
c) Two bunches of 6 cold cubes each were added.
d) Four bunches of 7 hot cubes each were removed.
e) Three bunches of 6 cold cubes each were removed.

2) Describe the action involving hot or cold cubes that is represented by each if the following arithmetic expressions and state how the temperature would change overall.

a) ${}^{+}4 - {}^{-}3$
b) ${}^{-}6 + {}^{-}4$
c) ${}^{-}10 \cdot {}^{-}5$
d) ${}^{+}4 \cdot {}^{-}8$

SHARE

After each number is made, have the students share the different ways they found the number 2, -2 or all the other numbers you needed them to make. Encourage them to check the other students work and to write the answers on the board so ALL students can see the possibilities.

SUMMARIZE

"Previously, you may have thought that 2 and -2 could only be written one way. Now you know that numbers can be written multiple ways and they still represent the same quantity. Tomorrow we are going to start cooking with the chefs and are going to make sure that the temperature is perfect my adding or taking away cubes"

Homework:

Explain each problem in terms of the model of hot and cold cubes.

Your explanation should describe the action and state how the temperature changes overall.

1) $^-6 + {}^-9$

2) $^-7 - {}^-10$

3) $^+5 \cdot {}^-2$

4) $^-4 - {}^+6$

5) $^+3 + {}^-7$

6) $^-6 \cdot {}^+9$

7) $^-3 \cdot {}^-4$

8) $^+8 - {}^-12$

9) $^-12 + {}^+5$

Lesson on Triangle and Square numbers

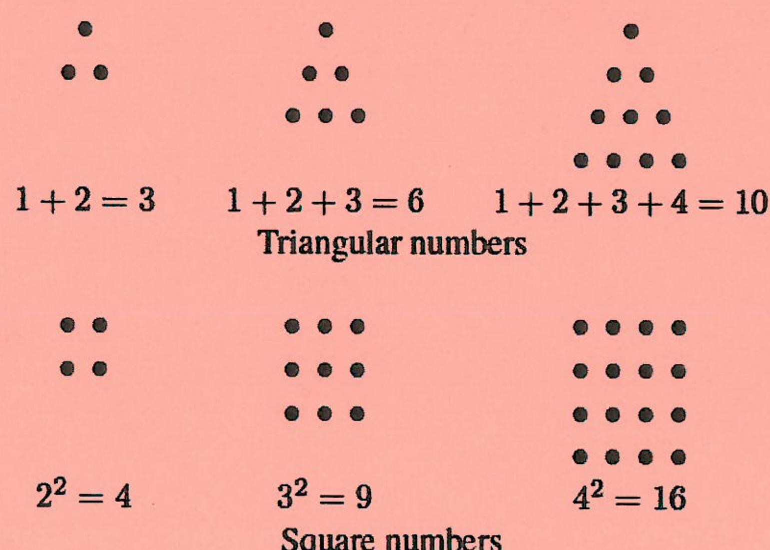
Goals:
Theoretic Questions
The main goal of number theory is to discover interesting and unexpected relationships between different sorts of numbers and to prove that these relationships are true. In this section we will describe a few typical number theoretic problems, some of which we will eventually solve, some of which have known solutions too difficult for us to include, and some of which remain unsolved to this day.
Sums

Launch:
Show students the first 3 or 4 triangle and then Square number on graph paper.
Questions
What are the patterns, Can you find a way to get the next number, What about the N? Number?
triangular 1; 3; 6; 10; 15; 21……square 1; 4; 9; 16; 25; 36; : : :
Can you draw the first 6 of each type? Numbers? Are there pentagonal, Hexagonal, ect.. Can you drawn or find them?

Explore:

Give out lab sheet with samples and place to graph with other Questions

Number Shapes. The square numbers are the numbers 1, 4, 9, 16, … that can be arranged in the shape of a square. The triangular numbers are the numbers 1, 3, 6, 10, … that can be arranged in the shape of a triangle. The first few triangular and square numbers are illustrated in Figure 1.1.



Now you need to graph these numbers. What do you see

Share

Summerize:

1.) Display a spreadsheet program and check for background knowledge of students,
2.) Have students enter the new data into the spreadsheets and create a graph of the information and discuss the new graphs. Are the graph discrete or continues ? Why?

Squares I. Can the sum of two squares be a square? The answer is clearly "YES"; for example 32 + 42 = 52 and 52 + 122 = 132. These are examples of Pythagorean triples. We will describe all Pythagorean triples in another lesson

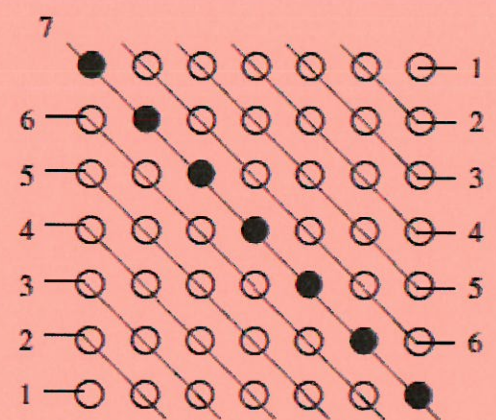
Numbers That Form Interesting Shapes
A natural question to ask is whether there are any triangular numbers that are also square numbers (other than 1). The answer is "YES," the smallest example being 36 = 62 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8: So we might ask whether there are more examples

finitely many? To search for examples, the following formula is helpful:

$$1 + 2 + 3 + \cdots + (n-1) + n = \frac{n(n+1)}{2}.$$

There is an amusing anecdote associated with this formula. One day when the young Carl Friedrich Gauss (1777–1855) was in grade school, his teacher became so incensed with the class that he set them the task of adding up all the numbers from 1 to 100. As Gauss's classmates dutifully began to add, Gauss walked up to the teacher and presented the answer, 5050. The story goes that the teacher was neither impressed nor amused, but there's no record of what the next make-work assignment was!

There is an easy geometric way to verify Gauss's formula, which may be the way he discovered it himself. The idea is to take two triangles consisting of $1 + 2 + \cdots + n$ pebbles and fit them together with one additional diagonal of $n+1$ pebbles. Figure 1.2 illustrates this idea for $n = 6$.



$$(1+2+3+4+5+6) + 7 + (6+5+4+3+2+1) = 7^2$$

To go to next level Question:

1.1. The first two numbers that are both squares and triangles are 1 and 36. Find the next one and, if possible, the one after that. Can you figure out an efficient way to find triangular–square numbers? Do you think that there are infinitely many?

1.2. Try adding up the first few odd numbers and see if the numbers you get satisfy some sort of pattern. Once you find the pattern, express it as a formula. Give a geometric verification that your formula is correct.

1.3. The consecutive odd numbers 3, 5, and 7 are all primes. Are there infinitely many such "prime triplets"? That is, are there infinitely many prime numbers p such that $p+2$ and $p+4$ are also primes?

1.4. It is generally believed that infinitely many primes have the form N^2+1, although no one knows for sure.

(a) Do you think that there are infinitely many primes of the form N^2-1?
(b) Do you think that there are infinitely many primes of the form N^2-2?
(c) How about of the form N^2-3? How about N^2-4?
(d) Which values of a do you think give infinitely many primes of the form N^2-a?

Lesson: 20 Question Game

Middle level through HS

Standard: Read, write, represent and compare positive and negative rational numbers, expressed as integers, fractions and decimals.

Objective:

This game is similar to 20 questions and can be adjusted to students' level of understanding of number theory. It provides students an opportunity to think about the many ways numbers can be described (odd/even, whole number, fraction less than or greater than one, prime number, multiple of *n*, rational, etc.). Students will apply a variety of mathematical concepts and skills to solve problems and use mathematical reasoning to determine whether a number fits a generalization. It requires students to use logical reasoning to identify a number—they narrow down choices based on the answers to questions they ask.

Launch: The instructor will begin with a standard game of 20 questions and introduce the idea of game involving numbers

The level of this lesson can be adjusted by limiting or expanding the range of the numbers (1 to 50, 1 to 1000, etc.), the number types (whole numbers, fractions, integers, etc.), and the number of questions asked.

Explore: Students will initially get into groups of 2-3. The first problems will start with the teacher leading the group and providing the answers. Once the students have a general understanding of the game they will have group to group contests to see how many questions it will be till their group guesses the answer. Groups should write down their questions and keep track of how many questions they ask. The questions must be phrased so that the answer is either yes or no.

Share: After the number has been identified, discuss the different types of questions asked. The teacher will ask students to brainstorm other questions they could ask. Play the game again. Compare the number of questions it took for students to identify the number in the first game with the number it took in the second game.

Summarize: Brainstorm questions that would be helpful to ask if the number were between 1 and 1000 ("Is the number greater than 500?"), if the number were a fraction ("Is the numerator less than the denominator?"), if the number were a decimal ("Does the number have more than one decimal place?"), and so on.

As an extension, change the range of the number or the number type, and continue playing. Also, you may want to have a student choose the number and answer the other students' questions.

Lesson: Perfect Numbers

Objective: Students will find all of the "perfect" numbers from 1-100 and look for patterns pertaining to these numbers. Students will also find "deficient" and "abundant" numbers from 1-100.

Standard: Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.

Launch: People have been searching for number patterns since ancient times. Mathematicians noticed that some numbers are equal to the sum of all of their factors (but not including the number itself).

For example, 6 is a number that equals the sum of its factors: 1 + 2 + 3 equal 6. Numbers like 6 that equal the sum of their factors are called "perfect" numbers. If the sum of the factors is less than the number itself, the number is "deficient". If the sum of the factors is more than the number itself, the number is "abundant".

Is your birth date a perfect number? Or a deficient number? Or an abundant number?

How many perfect numbers are there? From 1-100? Are there more deficient, perfect, or abundant numbers?

Explore: Students will form groups of 3 and list the factors or the numbers from 1-100 in a chart along with the sum of the factors and identify the perfect, deficient, and abundant numbers.

Share: Groups will begin by sharing their number of perfect, deficient, and abundant numbers between 1-100 and compare with other groups. Then groups will identify category each number belongs to, starting with 1 and working to 100.

Summarize: The instructor will lead a discussion during the sharing time to achieve a consensus from all groups. That consensus being that 1 is neither of the categories, there are 2 perfect numbers, 20 abundant numbers, and 77 deficient. Results as follows:

Perfect, Abundant and Deficient Numbers #1-100

Number	Factors	Sum	Category
2	1, 2	1	deficient
3	1, 3	1	deficient
4	1, 2, 4	3	deficient
5	1, 5	1	deficient
6	1, 2, 3, 6	6	perfect
7	1, 7	1	deficient
8	1, 2, 4, 8	7	deficient
9	1, 3, 9	4	deficient
10	1, 2, 5, 10	8	deficient
11	1, 11	1	deficient
12	1, 2, 3, 4, 6, 12	16	abundant
13	1, 13	1	deficient
14	1, 2, 7, 14	10	deficient
15	1, 3, 5, 15	9	deficient
16	1, 2, 4, 8, 16	15	deficient
17	1, 17	1	deficient
18	1, 2, 9, 18	12	deficient
19	1, 19	1	deficient
20	1, 2, 4, 5, 10, 20	22	abundant
21	1, 3, 7, 21	11	deficient
22	1, 2, 11, 22	14	deficient
23	1, 23	1	deficient
24	1, 2, 3, 4, 6, 8, 12, 24	36	abundant
25	1, 5, 25	6	deficient
26	1, 2, 13, 26	16	deficient
27	1, 3, 9, 27	13	deficient
28	1, 2, 4, 7, 14, 28	28	perfect
29	1, 29	1	deficient
30	1, 2, 3, 5, 6, 10, 15, 30	42	abundant
31	1, 31	1	deficient
32	1, 2, 4, 8, 16, 32	31	deficient

Name____________________

Number Theory Post Test

1. List the factors of 24.

   Is 24 a perfect, deficient or abundant number?

2. Give an example of 3 perfect numbers.

3. What is the property that all perfect numbers possess?

4. Give one example of the "golden ratio" existing in:
   a. Art
   b. Architecture
   c. Nature

5. Given the categories a. Prime, b. Composite, c. Natural, d. Whole, e. Integer, f. Rational, g. Irrational. For each number list all the categories that each number belongs to.
   1. 5
   2. 0.99
   3. .-1
   4. $\sqrt{10}$
   5. $\sqrt{9}$
   6. 0
   7. $\frac{3}{8}$

6. Describe 3 ratios from the human body that may possess the golden ratio.
   1.

   2.

   3.

7. Find the sums

-2+-3 $\frac{2}{3} + -\frac{1}{6}$ -6.9 + 2.56

8. Complete each number sentence

-49 ÷ (-7)=

-6*-23=

9. In a football game, one team make seven plays in the first quarter. The results of those plays in order: gain of 12 yards, gain of 2 yards, loss of 10 yards, loss of 1 yards, loss of 6 yards, gain of 15 yards, gain of 5 yards, loss of 5 yards.

a.) what is the overall gain (or lose) from all seven plays?

b.) What is the average gain(or lose) per play?